學校 　　　　　　　　　　　　　姓名 　　　　　　　　　　　　　　座號
School _____Name _____Seat No._____

- 比賽共有 14 題：題 1–2 是選擇題，題 3–14 是填空題，每題 5 分。請把答案填寫在右方空格內。

- 可用鉛筆或原子筆作答。

- 禁止使用任何類型之計算機或計算工具。

- There are 14 questions. Questions 1–2 are multiple choice, questions 3–14 are fill-in-the-blanks, and each of them is 5 marks. Fill in the correct answers of Questions 1–14 in the boxes on the right.

- You can write your answer with pencil or ball pen.

- Any electronic calculators or computing tools of any kinds are not allowed.

符號　Notations

- 如果 p、q、r 爲數，則
  (i) $pq = p \times q$ 代表這兩個數的乘積；
  (ii) $pqr = p \times q \times r$ 代表這三個數的乘積。。

- If p, q, r are three numbers, then
  (i) $pq = p \times q$ represents the product of these two numbers.
  (ii) $pqr = p \times q \times r$ represents the product of these three numbers.

- 正整數 p 稱爲素數(或者質數)，如果 $p > 1$ 且 p 恰有兩個正約數 1 及 p。
  A positive integer p is called a prime number, if $p > 1$ and p has only 2 positive divisors 1 and p.

1. 以下哪個數不是7的倍數？
   Which number in the following is not a multiple of 7?
   A. 8638   **B.** 10024   **C.** 17017   **D.** 22687   **E.** 42001
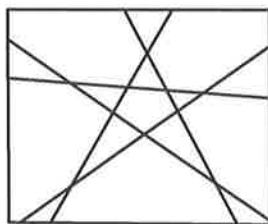
2. 桌上放著32張紙牌，甲、乙兩人輪流取走若干紙牌，規則如下：(i) 由甲先取；(ii) 每人每次至少取1張，至多取6張；(iii) 誰取走最後一張紙牌，誰就獲勝。甲為了保證獲勝，甲應該先取多少張紙牌？
   There are 32 cards on the desk. A and B play a game by taking turns to remove some cards from the desk. The rule of are as follows: (i) A removes some card(s) first; (ii) in each turn one can remove at least 1 card, and at most 6 cards; and (iii) the player who removes the last card is the winner. To make sure that A must win the game, how many cards should he take at first?
   A. 6   **B.** 5   **C.** 4   **D.** 3   **E.** 2

3. 如下圖所示，線段分割長方形為若干個不相交的區域，問長方形內有多少個區域？
   In the figure below, line segments divide the rectangle into several disjoint regions. How many regions are there inside the rectangles?

4. 計算 $0.4 \times \left[\dfrac{11}{52} \div 2\dfrac{3}{4} \times (4.3 - 1.8) \right] \times 26$
   Evaluate $0.4 \times \left[\dfrac{11}{52} \div 2\dfrac{3}{4} \times (4.3 - 1.8) \right] \times 26$

5. 整數 2023 共有 _____ 個正約數。
   The number of positive divisors of integer 2023 is _____.

6. 如果 $p \le q \le r$ 是三個素數(質數)，使得 $pq + qr + rp - 1$ 是 pqr 的倍數，求這三個素數之和 $p + q + r$。
   If $p \le q \le r$ are 3 primes numbers such that $pq + qr + rp - 1$ is a multiple of pqr, find the sum $p + q + r$.

7. 在 10 至 2023 之間的正整數中，有多少個數它的首位數碼與末位數碼之和等於 9？
   注意：18 是其中一個例子。
   Among all the positive integers between 10 and 2023, how many are there such that the sum of its first and last digits is equal to 9?
   Note that 18 is an example.

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1. 以下哪個數不是7的倍數？

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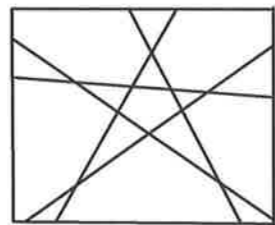
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3. 如下圖所示，線段分割長方形為若干個不相交的區域，問長方形內有多少個區域？

   In the figure below, line segments divide the rectangle into several disjoint regions. How many regions are there inside the rectangles?

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   If $p \le q \le r$ are 3 primes numbers such that $pq + qr + rp - 1$ is a multiple of pqr, find the sum $p + q + r$.

8. 如下圖所示，E、F 分別是三角形 ABC 的邊 AB、AC 上的中點。線 CE 與 BF 交於點 O。問：圖中面積相等的三角形有多少對？

   In the figure below, E, F are the mid-points of the sides AB, AC of triangle ABC respectively. Lines CE and BF meets at point O. How many pairs of triangles with equal areas are there in the figure?
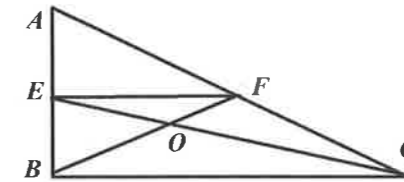
9. 把 1, 2, 3, 4, 5, 6, 7, 8, 9 分別填在以下的9個方格內，每個方格一個數且每個數不能重複，然後計算式子的結果 S。問結果 S 的最大值是多少？

   Fill in 1, 2, 3, 4, 5, 6, 7, 8, 9 in each of the boxes below, one number in a box, and no numbers are allowed to repeat. Then calculate the result S. What is the largest value of S?

   $$S = \boxed{}\boxed{}\boxed{} + \boxed{}\boxed{}\boxed{} - \boxed{}\boxed{}\boxed{}$$

10. 一個面積為1的黑色正三角形，每變換一次，即把每個黑色正三角形分為四個面積相等的小正三角形，中間的小正三角形塗上白色。下圖給出變換兩次後的圖形。問：經過五次變換後，所得的圖形中黑色區域的面積是多少？

    There is a black equilateral triangle with area 1. One performs the following operation: divide every black equilateral triangle into 4 smaller equilateral triangles with equal areas, and modify the color of the middle one from black to white. The following illustrates the figure undergoing 2 operations. After performing 5 operations, what is the area of black region in the resulting figure?
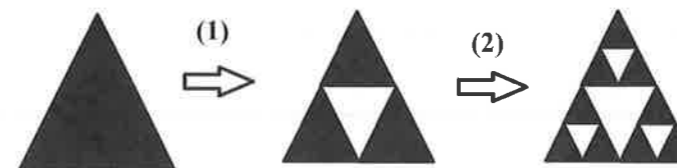
11. 定義 $f(1) = 7$，及對任意的正整數 n，$f(n + 1) = 7 \times f(n)$。
例如 $f(2) = 7 \times f(1) = 7 \times 7 = 49$。$f(2023)$ 最末的(個位)數碼是甚麼？
Define $f(1) = 7$, and $f(n + 1) = 7 \times f(n)$ for any positive integer n.
As an example, $f(2) = 7 \times f(1) = 7 \times 7 = 49$.
What is the last (unit) digit of $f(2023)$?

12. 甲、乙兩人同時從圓形跑道上同一起跑線前出發，沿順時針方向跑步，甲的速度比乙快。過一段時間，甲第一次從背後追上乙，這時甲立即背轉身子，以原來的時速沿逆時針方向跑去，當兩人再相遇時，乙恰好跑了 $\frac{3}{4}$ 圈。問：甲的速度是乙的多少倍？
Two persons A and B start running at the same time and at the same starting line on a circular track in the clockwise direction. The speed of A is faster than that of B. After a while, A catches up with B from behind for the first time, at that moment A turns his back and runs at the same speed in the counterclockwise direction. At the moment both A and B meet for the second time, B just finishes $\frac{3}{4}$ laps. What is the ratio of speed of A to that of B?

13. 100 位學生回答五道試題，有 81 位學生答對第一題，91 位學生答對第二題，85 位學生答對第三題，79 位學生答對第四題，74 位學生答對第五題。
問：在這 100 位學生中答對三道或三道以上的學生最小有多少個？
100 students take a test consisting of 5 questions. It is known that 81 students have correct answers in question 1, 91 students have correct answers in question 2, 85 students have correct answers in question 3, 79 students have correct answers in question 4, 74 students have correct answers in question 5. Among these 100 students, at least how many of them have 3 or more than 3 correct answers?

14. 如果你有 21 張卡片，上面分別寫著從 1 到 10 及 90 到 100 的所有數字，每張卡片只寫上一個數，且每個數都不能重複。在不能旋轉的情況下你應該如何將它們從左到右排列，以便在一行中得到最大可能的數字？
If you have 21 cards, filled with all the numbers from 1 through 10 and from 90 through 100. Eac' card is filled with only 1 number, and no number is allowed to repeat. How should you arrange the cards from left to right in order to create the largest possible number in a row if you can not rotate any cards?

由左至右,由上至下, 可分行寫出這 21 個數的排列次序。
You can write these 21 numbers starting from the left to right, from top to bottom in divided lines.