- 比賽共有有12題：題$1-2$是填空題，把答案填寫在右方空格内。題$3-12$是證明題，請把詳細的解題過程寫在背後的白紙上，並在左方角寫上題目編號。題$1-6$ 每題10 分，題$7-12$每題20分，

- There are 12 questions. Questions 1-2 are fill-in-the-blanks, write the answers in the box on the right. Questions 3-12 are proof-related, please write the arguments/steps of your solutions at the blank page with question number on the left-upper corner. Each of questions 1-6 is 10 marks, and each of questions 7-12 is 10 marks.

1. 若三角形的外接圓半徑爲2，這三角形的面積的最大值是_____

   If the radius of the circumcircle of a triangle is 2, then the maximium value of the area of the triangle is _____.

2. 從$0,1,2,3,4,5,6,7,8,9$這十個數中一次取出3個數，使得其和爲不小於10的偶數，則不同的取法有_____種。

   Each time pick three numbers from $0,1,2,3,4,5,6,7,8,9$ such that their sum is even and is not less than 10, there are _____ ways can you pick these three numbers.

(以下都是證明題, 請把解答過程寫在背後或空白的A4紙上)

3. 設O爲座標平面上的原點，一直線經過點$(3,4)$並交x-軸及y-軸分別於A及B兩點。已知$\triangle OAB$位於第一象限内，試確定$\triangle OAB$的面積的最小值，並給出理由。

   Let O be the origin in a coordinate plane. A line through point $(3,4)$ meets the x-axis and y-axis at points A and B respectively. If the $\triangle OAB$ is in the first quadrant, determine, with reason, the minimum value of the area of $\triangle OAB$.

4. 試確定$f(x)=x^3-3x-1$的最小值，其中x是任意的正數，並給出理由。
   Determine, with proof, the minimum value of $f(x)=x^3-3x-1$, where x is any positive number.

5. 設n是任意的正整數, 記$S_n=1^3+2^3+3^3+\cdots+n^3$。

   (i) 證明: $S_1=\left(\frac{1(1+1)}{2}\right)^2$。

   (ii) 如果$S_n=\left(\frac{n(n+1)}{2}\right)^2$，證明:$S_{n+1}=\left(\frac{(n+1)(n+2)}{2}\right)^2$。

   Let n be any positive integer, denote $S_n=1^3+2^3+3^3+\cdots+n^3$.

   (i) Prove $S_1=\left(\frac{1(1+1)}{2}\right)^2$.

   (ii) If $S_n=\left(\frac{n(n+1)}{2}\right)^2$, prove that $S_{n+1}=\left(\frac{(n+1)(n+2)}{2}\right)^2$.

6. 平面有5個點，其中任意3個點均不共線，以這些點爲端點連接線段。除這5個點外，試確定這些線段的交點個數，並給出證明。

   There are 5 points in a plane, no three of them are collinear. Join any pair of these points by a segment with the points as end-points. Determine, with proof, the number of intersection points of these segments other than the given 5 points.

7. 甲乙兩人進行如下遊戲，甲先開始，兩人輪流從$1, 2, \cdots, 100, 101$中每次任意勾去9個數，經過這樣11次勾掉後，還剩兩個數，這時所餘兩個數之差即爲甲的得分。試證明：不論乙怎樣做，甲有方法可至少取得55分。

A and B play the following game: A starts first, and then they take turns to remove 9 numbers from $1, 2, \cdots, 100, 101$, till there are only 2 numbers left, and the difference between the numbers left is the score of A. Prove that no matter how B plays, A has a way to score at least 55 marks.

8. 在圓周上任意寫上49個A及50個B，然後每輪進行下列兩項操作：

   (i) 在兩個相同字母之間寫上B，在兩個不同字母之間寫上A；

   (ii) 擦掉原有的字母並保留剛寫的字母。

   接著輪流繼續進行同樣的操作。問：能否經有限輪操作後使得圓周上的數字都變成B？給出理由。

   One writes 49 A's and 50 B's on a circle in any order. In each turn, one performs the following two operations:

   (i) insert letter B between any two identical adjacent letters, while insert letter A between distinct adjacent letters;

   (ii) then erase all original 59 letters, and keep the inserted ones.

   Continue performing the same operation in turns. Is it possible to turn all the letters to B after performing the operations finitely times. Explain your answer with reason.

9. 設 $x_1, x_2, \cdots, x_n$ 爲正數。求證以下不等式：
   Let $x_1, x_2, \cdots, x_n$ be positive, prove the following inequalities:

   (i) $(x_1 + \frac{1}{x_1})(x_2 + \frac{1}{x_2}) \geq (x_1 + \frac{1}{x_2})(x_2 + \frac{1}{x_1})$.

   (ii) $(x_1 + \dfrac{1}{x_1}) \cdots (x_{n-1} + \dfrac{1}{x_{n-1}})(x_n + \dfrac{1}{x_n}) \geq (x_1 + \dfrac{1}{x_2}) \cdots (x_{n-1} + \dfrac{1}{x_n})(x_n + \dfrac{1}{x_1})$.

10. 試確定(並給出證明)最小的實數C使得對滿足$\displaystyle\sum_{j=1}^{n} x_j = 1$的任意正實數$x_1, x_2, \cdots, x_n$，

    都有 $\displaystyle C\sum_{j=1}^{n} \frac{x_j^2}{1-x_j} \geq 1$。

    Determine, with proof, the minimum of real number C such that for any positive numbers $x_1, x_2, \cdots, x_n$ with $\displaystyle\sum_{j=1}^{n} x_j = 1$, the inequality $\displaystyle C\sum_{j=1}^{n} \frac{x_j^2}{1-x_j} \geq 1$ holds.

11. 求證：對任意的正數 x, y, z，以下不等式成立：

    Let x, y and z be any positive numbers, prove that

$$xyz(x+2)(y+2)(z+2) \leq \left(1 + \frac{2(xy+yz+zx)}{3}\right)^3.$$

12. 試找出所有能表成以下形式的整數：$\dfrac{b}{a} + \dfrac{c}{a} + \dfrac{c}{b} + \dfrac{a}{b} + \dfrac{a}{c} + \dfrac{b}{c}$，其中$a, b, c$爲兩兩互素的正整數。

    Find all positive integers that can be written in the following way $\dfrac{b}{a} + \dfrac{c}{a} + \dfrac{c}{b} + \dfrac{a}{b} + \dfrac{a}{c} + \dfrac{b}{c}$, where a, b, c are positive integers and they are pairwise relatively prime.