

- 比賽共有有 16 題：題 1-8 是填空題，每題 5 分，把答案填寫在右方空格內。題 9 – 16 是證明題，每題 10 分，請把詳細的解題過程寫在背後的白紙上，並在左方角寫上題目編號。
- There are 16 questions. Questions 1-8 are fill-in-the-blanks, each of them is 5 marks. Questions 9-16 are proof-related, each of them is 10 marks. Please write the arguments/steps of your solutions at the blank page with question number on the left-upper corner.

1. 計算 Evaluate

答案空格

$$\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \cdots + \frac{1}{\sqrt{2021} + \sqrt{2022}}.$$

- A. $\sqrt{2022} - 1$ B. $\sqrt{2022} + 1$ C. $\sqrt{2023} - 1$
 D. $\sqrt{2023} + 1$ E. 以上皆非 None of the above

2. 若正實數 x, y 滿足 $(\sqrt{x} + \sqrt{y})(\sqrt{x} + \sqrt{y} - 1) = 6$ ，則 $\sqrt{x} + \sqrt{y} = \underline{\hspace{2cm}}$ 。

If x, y are positive real numbers such that $(\sqrt{x} + \sqrt{y})(\sqrt{x} + \sqrt{y} - 1) = 6$, then $\sqrt{x} + \sqrt{y} = \underline{\hspace{2cm}}$.

- A. 1 B. 2 C. 3 D. 4 E. 以上皆非 None of the above

3. 已知 $a + b = 2, ab = -2022$ ，則 $a^2b + ab^2 = \underline{\hspace{2cm}}$ 。

If $a + b = 2$ and $ab = -2022$, then $a^2b + ab^2 = \underline{\hspace{2cm}}$.

- A. -4044 B. -2022 C. 2022 D. 4044 E. 以上皆非 None of the above

4. 已知有恆等式 $\frac{x-3}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ ，其中 A, B 是常數。求 $A - B = \underline{\hspace{2cm}}$ 。

If the identity $\frac{x-3}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ holds for some constants A and B , then $A - B = \underline{\hspace{2cm}}$.

- A. 1 B. 2 C. 3 D. 4 E. 以上皆非 None of the above

5. 若同時滿足以下不等式 $x - 1 > a$ 及 $x + 1 < b$ 的解集為 $1 < x < 3$ ，則 $a^2b^2 = \underline{\hspace{2cm}}$ 。

If $1 < x < 3$ is the solution set of the following 2 inequalities $x - 1 > a$ and $x + 1 < b$, then $a^2b^2 = \underline{\hspace{2cm}}$.

- A. 0 B. 1 C. 2 D. 3 E. 以上皆非 None of the above

6. 有 個整數 n 能使 $\frac{n^2+7}{n+3}$ 為整數。

There are integers n such that $\frac{n^2+7}{n+3}$ is an integer.

- A. 4 B. 6 C. 8 D. 10 E. 以上皆非 None of the above

7. 記 $\{x\} = x - [x]$ 其中 $[x]$ 為不超過 x 的最大整數，例如 $\{\frac{7}{3}\} = \frac{1}{3}$ 。

Denote by $\{x\} = x - [x]$ where $[x]$ the largest integer not exceeding x .

$$\left\{\frac{2021}{5}\right\} + \left\{\frac{2022}{5}\right\} + \cdots + \left\{\frac{4044}{5}\right\} = \underline{\hspace{2cm}}.$$

8. 分解因式 Factorize

$$x^2 - 2x - 2y^2 + 4y - xy = \underline{\hspace{2cm}}.$$

(以下都是證明題, 請把解答過程寫在背後或空白的A4紙上)

9. 已知 $a < b < c$, 確定 $\frac{1}{a-b} + \frac{1}{b-c} + \frac{1}{c-a}$ 的正負號, 並給出證明。

If $a < b < c$, determine, with proof, the sign of $\frac{1}{a-b} + \frac{1}{b-c} + \frac{1}{c-a}$

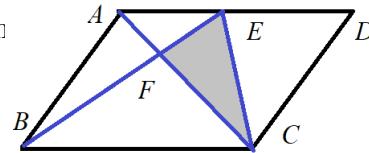
10. 從 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 這十個數中一次取出 3 個數, 使得其和為不小於 10 的偶數, 則不同的取法有多少種? 試給出理由

Each time pick three numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 such that their sum is even and is not less than 10, how many different ways can you pick these three numbers? Explain your answer with reasons.

11. 在右圖 $ABCD$ 是平行四邊形, $\angle B = 60^\circ$, E 是 AD 的中點, AC 交 BE 於點 F 。記三角形 EFC 及平行四邊形 $ABCD$ 的面積分別為 S_{EFC} 及 S_{ABCD} , 試確定比值 S_{EFC}/S_{ABCD} , 並給出理由。

As shown in the right figure, $ABCD$ is a parallelogram, $\angle B = 60^\circ$, E is the mid-point of AD , and AC meets BE at F . Let S_{EFC} and S_{ABCD} be the area of triangle EFC and parallelogram $ABCD$ respectively.

Determine, with reason, the ratio S_{EFC}/S_{ABCD} ,



12. 在 $\triangle ABC$ 中, $\angle B < \angle C$, $\angle A$ 的三等分線恰為 BC 邊上的中線及高, 試確定 $\angle C$, 並給出理由。

In $\triangle ABC$, $\angle B < \angle C$, and the 2 angle tri-sector of $\angle A$ are the median and altitude of $\triangle ABC$ respectively. Determine, with reason, the value of $\angle C$.

13. 試寫出 $2^{2022} - 1$ 的一個正因數 d 且 $10 < d < 99$, 並給出理由。

Find, with reason, a positive factor d of $2^{2022} - 1$ with $10 < d < 99$.

14. 平面有 5 個點, 其中任意 3 個點均不共線, 以這些點為端點連接線段。除這 5 個點外, 試確定這些線段的交點個數, 並給出證明。

There are 5 points in a plane, no three of them are collinear. Join any pair of these points by a segment with the points as end-points. Determine, with proof, the number of intersection points of these segments other than the given 5 points.

15. 設 x, y 是非負整數使得 $x + 2y$ 是 5 的倍數, $x + y$ 是 3 的倍數, 且 $2x + y \geq 99$, 試確定 $7x + 5y$ 的最小值, 並給出證明。

If x, y are non-negative integers such that $x + 2y$ is divisible by 5, $x + y$ is divisible by 3, and $2x + y \geq 99$, determine, with proof, the minimum value of $7x + 5y$.

16. 若三角形的外接圓半徑為 2, 試確定這三角形的面積的最大值, 並給出證明。

If the radius of the circumcircle of a triangle is 2, determine, with proof, the maximum value of the area of the triangle.