

姓名 Name _____ (in ID card)

學校 School _____ (No short form)

班級 Form _____

座位編號 Seat Number _____

此卷有 13 道題目：題 1–4 每題 5 分；題 5–11 每題 8 分；題 12–13 每題 12 分。

There are 13 questions in this paper: Questions 1 – 4 have 5 marks each;
Questions 5 – 11 have 8 marks each; Questions 12 – 13 have 12 marks each.

可用鉛筆、黑色或藍色的筆填寫。

You can write with pencil, black or blue pens.

手機號 Phone No _____

(可以不填手機號, 只為通知有關訓練及測試.)

not necessary to fill in phone no, just for passing information of training and test)

未有通知前不能翻閱試卷

Don't FLIP this paper

without consent.

1. 有多少互異的整數解組 (x, y, z) 同時滿足下列方程組

$$xy = z - x - y, \quad yz = x - y - z, \quad zx = y - z - x?$$

How many distinct triples (x, y, z) satisfy all 3 equations

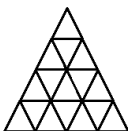
$$xy = z - x - y, \quad yz = x - y - z, \quad zx = y - z - x?$$

- A. 1 B. 2 C. 3 D. 4 E. 5

2. 右圖有多少個三角形?

How many triangles are there in the right figure?

- A. 25 B. 26 C. 27 D. 28.



3. 若 a, b 為正數且滿足 $12345 = (111 + a)(111 - b)$, 則以下正確的是:

Let a, b be positive and $12345 = (111 + a)(111 - b)$. Which is correct:

- A. $a > b$ B. $a = b$ C. $a < b$ D. $a^2 + b^2 < 2ab$.

4. 記 $D(n)$ 為正整數 n 的所有正因數 (約數) 之和, 如 $D(6) = 1 + 2 + 3 = 6$.

問有多少個正整數 n 使得 $D(n) = 42$?

Denote by $D(n)$ the sum of all positive divisors of n , for example $D(6) = 1 + 2 + 3 = 6$. How many integers n such that $D(n) = 42$?

- A. 1 B. 2 C. 3 D. 4

5. 設 P 為凸四邊形 $ABCD$ 形內的一點, X, Y, Z, T 分別為 $ABCD$ 四邊的中點.

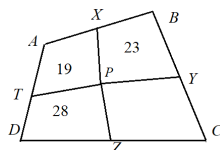
若四邊形 $DTPZ, AXPT, BYPX, CZPY$ 的面積分別為 28, 19, 23, S . 求 S .

P is a point in convex quadrilateral $ABCD$,

and X, Y, Z, T are midpoints of 4 sides of $ABCD$.

If the areas of $DTPZ, AXPT, BYPX, CZPY$

are 28, 19, 23, S respectively, find S .



$$S =$$

6. 計算 Evaluate $P = \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{2020^2}\right) \left(1 - \frac{1}{2021^2}\right)$.

$$P =$$

7. 設 a, b, c 為數滿足 $b = 2a + 1, c = 3b + 2, 120 - a = 4c$, 則

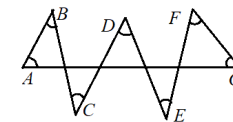
If a, b, c are numbers such that $b = 2a + 1, c = 3b + 2, 120 - a = 4c$, then

$$a + b + c =$$

8. 如下圖所示, As shown in the figure below

$$\angle A = 60^\circ, \angle B = 47^\circ, \angle D = 50^\circ,$$

$$\angle F = 48^\circ, \angle G = 57^\circ, x = \angle C = \angle E.$$



$$x =$$

9. 在直角三角形 ABC 中, $\angle C = 90^\circ, 3a + 2b = 2S$, 其中 S 是 $\triangle ABC$ 的面積, $a = BC, b = AC$. 試求 S 的最小值.

In a right-angle triangle $ABC, \angle C = 90^\circ$, and $3a + 2b = 2S$, where $a = BC, b = AC$, and S is the area of ABC . Then the minimum value of S is _____.

10. 從 $1, 2, \dots, 130$ 中任取 n 個數, 使得其中必定有兩數 x, y 滿足 $1 < \frac{x}{y} \leq \frac{3}{2}$, 則 Choose any n numbers among $1, 2, \dots, 130$, such that there are 2 integers x, y satisfying $1 < \frac{x}{y} \leq \frac{3}{2}$. Then

n 至少是

n is at least

11. 設 α, β 為 $x^2 + 2x - 6 = 0$ 的兩個互異根, 試寫出根為 $\frac{1}{\alpha-1}, \frac{1}{\beta-1}$ 的二次方程。

Let α, β be two distinct roots of $x^2 + 2x - 6 = 0$, find the quadratic equation with roots $\frac{1}{\alpha-1}, \frac{1}{\beta-1}$.

請把以下兩道證明題的答案寫在空白的 A4 紙上

12. 設 a, b, c 為正數, 求證 $(a + b + c)^3 \geq 27abc$ 。

If a, b and c are positive, prove that $(a + b + c)^3 \geq 27abc$.

13. 設 S 為凸四邊形 $ABCD$ 的面積, 求證: $2S \leq AB \times CD + BC \times AD$.

Let $ABCD$ be a convex quadrilateral with area S . Prove that

$$2S \leq AB \cdot CD + BC \cdot AD.$$