

姓名 Name _____ (in ID card)

學校 School _____ (No short form)

班級 Form _____

座位編號 Seat Number _____

此卷有 15 道題目：

題 1 – 4 道是選擇題，每題 4 分，只須在直線上的空位填寫英文字母：

A, B, C, D, E;

題 5 – 8 是填空题，每題 7 分，只須填寫正確答案，不須填寫過程。

題 9 – 15 是證明題，每題 15 分，必須填寫完整的計算或證明。

There are 15 questions in this paper:

The questions 1-4 are multiple-choice, 4 marks each. Fill in A, B, C, D, E in the underlined space provided.

The questions 5 – 8 are fill-in-blanks, 7 marks each. Fill in final answers, and no steps are needed.

The last 9 – 15 are long questions requiring written proofs in your answers, 15 marks each.

可用鉛筆、黑色或藍色的筆填寫。

You can write with pencil, black or blue pens.

手機號 Phone No _____

(可以不填手機號，只為通知有關訓練及測試)
not necessary to fill in phone no, just for passing information of training and test)

同學注意事項：

- 首先把手機關掉。比賽期間被發現手機未關掉的同學將被取消比賽資格。
- 把身份證明文件如學生證置於桌上右上角。若未能提供身份證明文件，請立即舉手示意工作人員。
- 現在填寫在試卷的左上方有關姓名、學校、班級、座位編號等空格上的資料。在比賽期間若需要前往洗手間請先舉手，等候工作人員指示。
- 在司儀宣佈“比賽開始”指令前，同學請勿翻開試卷。
- 在司儀宣佈“比賽開始”指令後，同學才可以作答。
- 當司儀宣佈“比賽結束”指令後，同學必須立刻停止作答，把(已經寫上姓名的)附加答題紙放入考卷內等候工作人員收取；其餘的紙張會當作草稿紙，其內容不會被評分。

Attention:

- First turn off your mobile phone. You will be disqualified if your phone is found switched on during the test.
- Put your identification document (student card or ID card) at the upper right hand corner of your desk. Raise your hand, if you could not provide any ID.
- Fill in your name, school, form and seat no. on the left of this paper. During the test, raise your hand if you want to go to toilet and wait for further instruction.
- Before a signal “TEST BEGINS” is announced, you are not allowed to turn any page of the paper.
- After a signal “TEST BEGINS” is announced, you can then answer the questions.
- Once a signal “TEST STOPS” is announced, you must stop answering, put any additional answer sheets (with your name) inside the test paper, and wait for the working crew to collect your test paper; the other papers will be treated as rough work, and they will be not be marked.
- During collecting the test papers, you must keep silent at your seat, and wait for further instruction. Once a specified signal “CAN LEAVE” is announced, you can then follow the instruction to leave the hall in order.

1. $\frac{109}{9990}$ 的循環小數表示是 The decimal representation of $\frac{109}{9990}$ is _____.

A. 0.109 B. 0.019 C. 0.0109 D. 0.0109 E. None of the above 以上皆非

2. 設 N 為滿足方程 $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 1$ 的有序三元正整數組 (x_1, x_2, x_3) 的個數，則 Let N be the number of ordered triples (x_1, x_2, x_3) of positive integers such that

$$\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 1. \text{ Then}$$

A. 1 B. 3 C. 5 D. 7 E. None of the above 以上皆非

$$N = \underline{\hspace{2cm}}.$$

3. 在所有小於 1 的有理數之中，記 S 為約簡後分母是 2019 的分數之和，則 In all the rational numbers smaller than 1, let S be the sum of those reduced

fractions with denominator 2019. Then $S = \underline{\hspace{2cm}}.$

A. 1 B. 3 C. 673 D. 2019 E. None of the above 以上皆非

4. n 對夫妻參加同一聚會，每個男士與每一個人握手（但不包括自己的妻子），女士之間不會握手。聚會中共有 S 次握手，則 n couples showed up in a gathering, every man shook hands with everyone except his wife, and ladies had no hand-shaking with each others. Let S be the number

of hand-shakings. Then

$$S = \underline{\hspace{2cm}}$$

A. $\frac{n(n-1)}{6}$ B. $\frac{n(n-1)}{5}$ C. $\frac{n(n-1)}{4}$ D. $\frac{n(n-1)}{3}$

E. None of the above 以上皆非

5. 設 $A(0, 4), B(2, 2)$ 為橢圓 $E: \frac{x^2}{9} + \frac{y^2}{25} = 1$ 內的兩點， M 為橢圓 E 上的任一點。記 m 為 $|AM| + |MB|$ 的最小值，則 Let m be $|AM| + |MB|$ the minimum value, then

Let $A(0, 4), B(2, 2)$ be two points inside the ellipse $E: \frac{x^2}{9} + \frac{y^2}{25} = 1$, M be a point on the ellipse E , and m be the minimum value of $|AM| + |MB|$. Then

$$m = \underline{\hspace{2cm}}.$$

6. 滿足 $P(x)^2 - 1 \equiv 4P(x^2 - 4x + 1)$ 的實係數多項式 $P(x)$ 是

All polynomials $P(x)$ of real coefficients satisfying $P(x)^2 - 1 \equiv 4P(x^2 - 4x + 1)$

are _____.

7. 已知正實數 x, y 滿足 $x^3 + y^3 \leq 2$ ，記 M 為 $x+y$ 的最大值，則 Let x, y be positive reals such that $x^3 + y^3 \leq 2$, and M be the maximum value of

$$x + y, \text{ then } M = \underline{\hspace{2cm}}$$

8. 列出滿足以下方程的所有正整數對 (x, y) : $(x, y) = \underline{\hspace{2cm}}$

Determine all pairs (x, y) of positive integers such that

$$x^{2^x} = y^{512^y}.$$

9. 試求最小的正整數 n ，使得在任意 n 個不同的整數中必存在 3 個 $a < b < c$ 且 $3 \mid (ab + bc + ca)$ 。

Determine the least positive integer n such that in every set consisting of n integers there are a, b and c in the set such that $a < b < c$ and $3 \mid ab + bc + ca$.

10. 定義數列 $(u_n)_{n \geq 1}$: $u_1 = 4$ 及對任意 $n \geq 1$ 有 $u_{n+1} = 2u_n + \sqrt{3u_n^2 + 1}$ 。

Define a sequence $(u_n)_{n \geq 1}$ by $u_1 = 4$ and $u_{n+1} = 2u_n + \sqrt{3u_n^2 + 1}$ for all $n \geq 1$. 求 : $S_n = u_{n+2} - 4u_{n+1} + u_n$ 當 $n \geq 1$ 。

Find $S_n = u_{n+2} - 4u_{n+1} + u_n$ for all $n \geq 1$.

11. 確定正整數 S 的個位數，並說明理由。

Find, with reason, the unit-digit of the positive integer

$$S = (2 + \sqrt{3})^{2019} + (2 - \sqrt{3})^{2019}.$$

12. 確定所有正整數 n ，使得 $S_n = 2^8 + 2^n$ 是完全平方數，並說明理由。

Determine, with reason, all positive integer(s) n such that $S_n = 2^8 + 2^n$ is the square of an integer.

13. 如圖 $\odot O$ 是 $\triangle ABC$ 的旁切圓，切點為 D, E, F 。若 R 是 $\odot O$ 的半徑，求證：

As shown in the figure, $\odot O$ is the A-excircle of $\triangle ABC$, touching the sides the lines BC, BA and AC at points D, E and F . If R is the radius of $\odot O$, prove that

$$\frac{2R}{BC} \leq \frac{1 + \sin(\angle A/2)}{\cos(\angle A/2)}.$$



