

姓名 Name _____

學校全名 School _____ (No short form)

班級 Form _____

座位編號 Seat Number _____

此卷有 13 道題目：

首 6 道是選擇題，每題 4 分，只須在直線上的空位填寫英文字母：A, B, C, D, E。
題 7—9 是填充題，每題 7 分，只須填寫正確答案，不須填寫過程。

題 10—13 是證明題，每題 15 分，必須填寫完整的計算或證明。

There are 14 questions in this paper:

Questions 1—6 are multiple choice, 4 marks each, only fill in English letters:

A, B, C, D, E in the underlined space provided.

Questions 7—9 are fill-in-blanks, 7 marks each, only fill in correct answer.

Questions 10—13 are long questions, 15 marks each, complete steps or proofs are required.

可用鉛筆、黑色或藍色的筆填寫。

You can write with pencil, black or blue pens.

手機號 Phone No _____

(可以不填手機號，只為通知有關訓練及測試)

Not necessary to fill in phone no, just for passing information of training and test)

同學注意事項：

- 首先把手機關掉。比賽期間被發現手機未關掉的同學將被取消比賽資格。
- 把身份證明文件如學生證置於桌上右上角。若未能提供身份證明文件，請立即舉手示意工作人員。
- 現在填寫在試卷的左上方有關姓名、學校、班級、座位編號等空格上的資料。在比賽期間若需要前往洗手間請先舉手，等候工作人員指示。
- 在司儀宣佈“比賽開始”指令前，同學請勿翻開試卷。
- 在司儀宣佈“比賽開始”指令後，同學才可以作答。
- 當司儀宣佈“比賽結束”指令後，同學必須立刻停止作答，把(已經寫上姓名的)附加答題紙放入考卷內等候工作人員收取；其餘的紙張會當作草稿紙，其內容不會被評分。

Attention:

- First turn off your mobile phone. You will be disqualified if your phone is found switched on during the test.
- Put your identification document (student card or ID card) at the upper right hand corner of your desk. Raise your hand, if you could not provide any ID.
- Fill in your name, school, form and seat no. on the left of this paper. During the test, raise your hand if you want to go to toilet and wait for further instruction.
- Before a signal “TEST BEGINS” is announced, you are not allowed to turn any page of the paper.
- After a signal “TEST BEGINS” is announced, you can then answer the questions.
- Once a signal “TEST STOPS” is announced, you must stop answering, put any additional answer sheets (with your name) inside the test paper, and wait for the working crew to collect your test paper; the other papers will be treated as rough work, and they will be not be marked.
- During collecting the test papers, you must keep silent at your seat, and wait for further instruction. Once a specified signal “CAN LEAVE” is announced, you can then follow the instruction to leave the hall in order.

選擇題：只須填寫 A, B, C, D, E

Multiple Choice Questions: Fill in A, B, C, D, E.

1. 9 個連續正整數最多有 x 個素數；9 個連續正整數最少有 y 個素數。則

There are at most x primes and at least y primes in 9 consecutive positive integers.

Then $(x, y) =$ _____

- A. (3, 1) B. (3, 0) C. (2, 1) D. (2, 0) E. None of the above 以上皆非

2. 設三個分數的和是 $\frac{1}{4}$ ，約簡後它們各自分母、各自分子的含比按先後次序為 1 : 2 : 3、4 : 5 : 6。這三個分數中最小的是 M 。則

The sum of 3 fractions is $\frac{1}{4}$. After simplifying the fractions, their denominators, and their numerators in the same order are in ratios 1 : 2 : 3 and 4 : 5 : 6 respectively. Let M be the smallest of the 3 fractions. Then

$M =$ _____

- A. $\frac{1}{22}$ B. $\frac{1}{23}$ C. $\frac{1}{24}$ D. $\frac{1}{25}$ E. 以上皆非 (None of the above)

3. 記 S 為 $(10^{2019} - 9) \times 10^2$ 在十進制下的數字和，則

Let S be the sum of the digits of $(10^{2019} - 9) \times 10^2$ in decimal, then

$S =$ _____

- A. 18161 B. 18162 C. 18163 D. 18164 E. 以上皆非 (None of the above)

4. 設正整數 a, b 滿足 $104 < a + b < 108$ 及 $0.91 < \frac{a}{b} < 0.92$ ，則

If positive integers a and b satisfy $104 < a + b < 108$, and $0.91 < \frac{a}{b} < 0.92$. Then $21a + 23b =$ _____.

- A. 2019 B. 2109 C. 2901 D. 2910 E. 以上皆非 (None of the above)

5. 設 $s = \frac{1}{3-\sqrt{7}}, t = s - [s]$ ，其中 $[s]$ 是不大於 s 的最大整數，則

Let $s = \frac{1}{3-\sqrt{7}}, t = s - [s]$, where $[s]$ is the largest integer not exceeding s , then

$t^2 + t =$ _____

- A. 0 B. $\frac{1}{2}$ C. 1 D. $\frac{3}{2}$ E. 以上皆非 (None of the above)

6. 設 $\triangle ABC$ 的邊長為 a, b, c 滿足 $a(a+b+c) = b(a+b)$ ，則

Let a, b, c be the lengths of sides of $\triangle ABC$ satisfying $a(a+b+c) = b(a+b)$, then

- A. $\angle B > 2\angle A$ B. $\angle B = 2\angle A$ C. $\angle B < 2\angle A$ D. $\angle B = \angle C$
E. 以上皆非。

填充題 (只要答案) Fill in the blanks (No Steps)

7. 計算以下的乘積 P : Evaluate the product P : $P =$ _____

$$P = \left(1 - \frac{1}{2 \times 2}\right) \left(1 - \frac{1}{3 \times 3}\right) \cdots \left(1 - \frac{1}{2018 \times 2018}\right) \left(1 - \frac{1}{2019 \times 2019}\right)$$

8. 甲、乙、丙、丁四人的平均年齡 S 是整數且 $S > 30$ ，如果甲的年齡是乙的伍份

之四，丙的年齡是乙的三份之二，丁比甲大 1 歲，則

The average age S of Ann, Ban, Cat, Ding is an integer and $S > 30$. If Ann's age is $\frac{4}{5}$ of Ban's, Cat's age is $\frac{2}{3}$ of Ban's, and Ding is 1 year older than Ann, then

$S =$ _____.

9. 設 $(1 - x^2 - x^3)^3 = 1 + a_1x + a_2x^2 + \cdots + a_9x^9$ 的展開式，其中 a_i 為項 x^i 的係數。則

Let $(1 - x^2 - x^3)^3 = 1 + a_1x + a_2x^2 + \cdots + a_9x^9$ be the expansion, where a_i is the coefficient of the term x^i , then

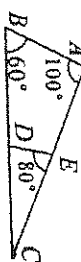
$a_1 + a_2 + a_3 + a_4 + a_5 =$ _____

10 ~ 13
 題 11-14 需要寫下詳細解題過程

10 ~ 13

Steps are required in the following Problems 11-14

10. 記 $S_{XYZ} = \triangle XYZ$ 的面積。如圖在 $\triangle ABC$ 中， E 是 AC 的中點， D 是 BC 邊上的一點。若 $BC = 1$ ， $\angle BAC = 100^\circ$ ， $\angle CED = 80^\circ$ ，則



Let S_{XYZ} = area of $\triangle XYZ$. In figure, E is midpoint of AC , and D is a point on side BC . If $BC = 1$, $\angle BAC = 100^\circ$, $\angle CED = 80^\circ$, $\angle ABC = 60^\circ$, then

$$S_{ABC} + 2S_{CDE} = \underline{\hspace{2cm}}.$$

11. 計算以下的和 S ，其中 $[x]$ 代表不大於 x 的最大整數。證明你的答案是正確。

Evaluate the sum S , where $[x]$ is the largest integer not exceeding x . Prove that

your answer is correct.

$$S = \left[\frac{305 \times 1}{503} \right] + \left[\frac{305 \times 2}{503} \right] + \dots + \left[\frac{305 \times 502}{503} \right] = \underline{\hspace{2cm}}.$$

12. 能否用 25 個 1×4 的長方塊來完全覆蓋 10×10 的棋盤？說明理由。

Is it possible to cover a 10×10 chessboard with 25 pieces of 1×4 rectangles?

Give your reason.

$$13. S = \sqrt{\frac{1}{(a-b)^2} + \frac{1}{(b-c)^2} + \frac{1}{(c-a)^2}}.$$

若 a, b, c 為兩兩不等的有理數，問 S 是否有理數？說明理由。

Let a, b, c be pairwise distinct rational numbers. Is S rational? Give your reason.