

## **The Requirements of Basic Academic Attainments for Junior Secondary Mathematics**

### **I. Basic Rationale**

Mathematics is a science that studies the space forms and quantity relations. Its contents, thinking, methods and languages are important components of human culture. Mathematics is closely related to human activities. Being a scientific language and an effective tool, mathematics has exerted inestimable influence on the process of human development.

Mathematical literacy is the basic literacy necessary for every citizen in modern society. The Mathematics curriculum should promote all-around, sustained and harmonious development of students; help them to master the basic mathematics knowledge and skills necessary for social life adaptation and further development; equip them with the capability to use junior secondary mathematics knowledge for practical application, reasoning and communication. On these grounds, the Requirements of Basic Academic Attainments for Junior Secondary Mathematics shall comply with the following fundamental principles:

1. The mathematics curriculum should meet the individual needs of students, lay a foundation for their development.

The content of mathematics curriculum should not only focus on the development and application of mathematics, satisfying students' needs for continuous learning, work and everyday life, but also be close to students' reality, follow their psychological law of mathematics learning, as well as cater to the personality development needs of students. Besides, the content of mathematics curriculum not only covers the development and generation process of knowledge, but also promotes the development of each student in terms of emotions and attitudes, activity experience, and thinking ability. Through presenting the contents in different ways, the diverse learning needs of students can be satisfied, thus lay the foundation for their all-round and sustained development.

2. Through diverse learning methods, students' comprehension of mathematics knowledge is strengthened.

Mathematics learning is the process of discovering new knowledge based on previous knowledge and experience. Effective mathematics learning activities cannot simply rely on imitation and memorisation; mathematical experiments, investigations and exchanges are also important modes of mathematics learning. The focus of mathematical learning is to understand the relationship among mathematical knowledge and apply the acquired knowledge to solve practical problems.

3. Mathematics teaching should reflect the process of mathematical thinking and promote the common development of teachers and students.

Mathematics teaching is the teaching of mathematical thinking activities; it is the process of interaction and mutual development between teachers and students as well as among students. Through mathematics teaching, students not only acquire knowledge and skills, understand ideas and methods, but also go through the thinking process, learn how to think mathematically, improve their mathematical literacy and realise the social and cultural values of mathematics.

Mathematics teaching should be based on the learning needs of students, helping them to learn to observe the world, discover and raise questions, analyse and solve problems from a mathematical perspective as well as summarise and synthesise what has been learned, promote mathematical results, and eventually review and reflect on the learning process, so as to concretise the application of mathematics.

4. Attaching importance to the integration of mathematics and information technology

The development of information technology has had significant impact on the presentation of teaching contents, the learning style of students, teaching styles of teachers and ways of teacher-student interaction. Modern information technology is not only an important tool for exploring and presenting mathematical knowledge, but also a powerful tool that students can use to learn mathematics and solve problems. The design and implementation of mathematics curriculum should promote the appropriate and reasonable use of modern information technology; enhance the integration of information technology and the curriculum; develop rich learning resources; enhance students' awareness on and practical ability in using information technology to solve mathematical problems, thus realise the coordinated development of their information technology literacy as well as mathematics literacy.

## **II. Curriculum Goals**

1. Enable students to acquire important mathematics knowledge, basic mathematics thoughts and essential application skills which are necessary for social life, practical activities and their continuous study of mathematics or mathematics related disciplines.
2. Cultivate students' abilities in mathematical reasoning, calculation, spatial imagination and problem solving, as well as the abilities of cooperation, exchange and independent thinking.
3. Develop students' awareness on and ability in flexibly applying mathematical thinking to judge problems through participating in such mathematical activities as observation, practice, thinking, exploration, exchanges, etc.
4. Nurture students to recognise the value of mathematics; develop good study habits and scientific attitudes; experience the fun of mathematical activities and the advantages of using mathematical thinking and methods in solving certain problems.

5. Allow students to experience success in imaginative and creative activities; temper their willpower to overcome adversities; and become confident and creative learners.

### **III. The Requirements of Basic Academic Attainments in different learning domains.**

Explanation of coding:

- (1) The capital English letters represent the requirements of basic academic attainments in different learning domains; A – Numbers and Algebra, B – Shapes and space, C - Statistics and probability, D – Emotions, attitudes and values;
- (2) The first number following the English letter represents the serial number of learning categories in the respective learning domain.
- (3) The second number following the English letter represents the serial number of the requirement of basic academic attainments in the respective learning category.

#### **Learning domain A: Number and Algebra**

##### **Learning category A-1: Number and expression**

- A-1-1 Understand the meaning of negative numbers and be able to use them to represent quantities in daily life.
- A-1-2 Understand the definition of rational numbers; make use of the number line to understand the meanings of opposite numbers and absolute values; be able to find the opposite numbers and absolute values of rational numbers as well as compare the size of rational numbers.
- A-1-3 Master the calculation of addition, subtraction, multiplication, division, power, and simple mixed operations of rational numbers that can be solved in three steps; be able to use operations of rational numbers to solve simple practical problems.
- A-1-4 Be able to perform mixed operations with rational numbers; understand the operation rules of rational numbers.
- A-1-5 Understand scientific notation and the notion of approximate numbers; be able to perform approximate calculation; be able to find the approximate value of a number within three steps by rounding off method according to the required precision.
- A-1-6 Understand the definitions of square roots, arithmetic square roots and cube roots; be able to use the radical sign to represent the square root, arithmetic square root and cube root of a number.
- A-1-7 Understand the fact that powers and roots are inverse operations; be able to use the square root operation to find the square root of a non-negative number; be able to use the cube root operation to find the cube root of a number; be able to use a calculator to find the square root and cube root of a number.
- A-1-8 Understand the concept of irrational number and real number.

- A-1-9 Understand the definitions of the opposite numbers and absolute values of real numbers; know the one-one correspondence relationship between real numbers and points on the number line.
- A-1-10 In real contexts, realise the meaning of using a letter to represent a number.
- A-1-11 Understand the concept of algebraic expressions and the value of algebraic expressions; be able to express simple quantitative relationships by using algebraic expressions and find the value of an algebraic expression by using the substitution method.
- A-1-12 Understand the definition and basic properties of integer exponent powers.
- A-1-13 Understand the concept of integral expressions.
- A-1-14 Master the method of combining like terms; be able to remove and add brackets; be able to perform simple addition and subtraction of integral expressions; grasp the rules for multiplication of monomials, multiplication between monomials and polynomials as well as multiplication of polynomials; master the rules for division of monomials as well as dividing a polynomial by a monomial.
- A-1-15 Understand the formula for the difference of squares and perfect square formula; be able to use them to perform operations.
- A-1-16 Be able to use common factor extraction method, the formula for the difference of two squares and perfect square formula to carry out factorisation.
- A-1-17 Understand the concept of fractions by using real examples; grasp the concept of irreducible fractions; master the basic properties of fractions; be able to reduce fractions to lowest terms or to a common denominator.
- A-1-18 Master the operation rules of addition, subtraction, multiplication, and division of fractions; be able to perform simple fraction operations.
- A-1-19 Understand the concept of quadratic radicals and simplest quadratic radicals; master the operation rules of addition and subtraction as well as multiplication and division of quadratic radicals; be able to rationalise a denominator containing a radical and perform simple mixed operations of quadratic radicals.

### **Learning category A-2 Equation and inequality**

- A-2-1 Understand the basic properties of equations.
- A-2-2 Understand a linear equation in one unknown and its related concept.
- A-2-3 Learn the common methods for solving a linear equation in one unknown; be able to solve a linear equation in one unknown with numerical coefficients.
- A-2-4 Be able to solve simple problems by creating a linear equation in one unknown in accordance with the equivalence relations in the context of the concrete problem; and examine whether the result is reasonable or not based on the actual meaning of the concrete problem.
- A-2-5 Understand the concept of fractional equations; master the solution of fractional equations that can be reduced to linear equations in one unknown; understand the concept of extraneous roots and be able to examine whether or not a number is an extraneous root of a fractional equation.
- A-2-6 Be able to solve simple application problems by creating a fractional equation which can be reduced to a linear equation in one unknown.

- A-2-7 Understand the concept of the system of linear equation in two unknowns and those of linear equation in three unknowns, as well as their solution sets.
- A-2-8 Master the methods of elimination by substitution and elimination by addition or subtraction; be able to solve systems of linear equations in two unknowns and know their graphical solutions.
- A-2-9 Be able to create and solve a linear equation in two unknowns in accordance with the quantitative relationships in a concrete problem and examine whether the result is reasonable or not based on the actual meaning of the concrete problem.
- A-2-10 Understand the concept of a quadratic equation in one unknown.
- A-2-11 Be able to use the method of completing the square, the formula method and factorisation to solve quadratic equations in one unknown with numerical coefficients; be able to find approximate solution of a quadratic equation in one unknown by the graphical method.
- A-2-12 Be able to create and solve quadratic equations in one unknown in accordance with concrete problems and examine whether the result is reasonable or not based on the actual meaning of the concrete problem.
- A-2-13 Understand the concept and basic properties of inequality.
- A-2-14 Be able to solve a simple linear inequality in one unknown with numerical coefficients and indicate the solution set on the number line.
- A-2-15 Understand the concept of the system of a linear inequality in one unknown and its solution set; know how to find the solution of the systems of inequalities with the help of the number line.

### **Learning category A-3: Function**

- A-3-1 Understand the definitions of constants and variables.
- A-3-2 Understand the definitions of the range of independent variables and the function values; be able to find the range of independent variables and the simple function values.
- A-3-3 Understand the concept of functions and the three ways to represent a function; be able to give real examples of functions.
- A-3-4 Be able to graph functions by plotting point.
- A-3-5 Know the notions of directly proportional functions and linear functions; be able to determine the analytic form of these functions based on the conditions given in the question.
- A-3-6 Recognise the properties of directly proportional functions and linear functions; be able to graph them and indicate, according to the graph, how the function values change according to the change in independent variables.
- A-3-7 Be able to determine the analytic form of a linear function by using the method of undetermined coefficients.
- A-3-8 Be able to solve practical problems with linear functions.
- A-3-9 Understand the concept of inversely proportional functions and be able to determine their analytic form based on the conditions given in the question.
- A-3-10 Understand the properties of inversely proportional functions; be able to graph them and indicate, according to the graph, how the values of functions change according to the change in independent variables.

- A-3-11 Be able to solve practical problems by applying inversely proportional functions.
- A-3-12 Understand the concept and properties of a quadratic function; be able to graph it by plotting points; be able to determine its analytic form by using the method of undetermined coefficients.
- A-3-13 Be able to apply a formula to determine the vertex and the axis of symmetry of a quadratic function; be able to use the method of completing the square to determine the vertex and the axis of symmetry of a quadratic function.
- A-3-14 Be able to solve practical problems by using quadratic equations.

## **Learning domain B: Shapes and space**

### **Learning category B-1: Points, lines, planes, and angles**

- B-1-1 Understand points, lines, planes and geometries.
- B-1-2 Understand the meaning of the distance between two points; know how to measure the distance between two points.
- B-1-3 Master the axiom of straight line: two points determine a line; the line segment between two points is the shortest.
- B-1-4 Master the concept of the ray, the line segment, and the length of a line segment; be able to compare the length of line segments; know the meaning of sum and difference of line segments as well as the midpoint of a line segment.
- B-1-5 Intuitively understand the relationship between two non-coincident lines in the same plane: intersecting and parallel.
- B-1-6 Understand the concept of angle, angle size, angle measurement, sum and difference of angles.
- B-1-7 Know about straight angles and round angles; understand the size relationship between round angles, straight angles, obtuse angles, right angles and acute angles; master the classification of angles.
- B-1-8 Understand the concept of complementary angles, supplementary angles, adjacent angles and vertical angles.
- B-1-9 Combining with the living situation, know the parallel and intersecting relationships between any pair of non-coincident straight lines in a plane.
- B-1-10 Understand the concept of perpendicular line and perpendicular line segment; be able to draw a line perpendicular to a given line, passing through a given point by using a set square, compass or protractor.
- B-1-11 Understand the meaning of the distance of a point to a straight line; be able to measure the distance from a point to a straight line.
- B-1-12 Comprehend the concept of parallel lines, corresponding angles, alternate interior angles, and same-side interior angles.
- B-1-13 Master the axiom of parallel lines: When two straight lines are cut by a transversal, if the corresponding angles are equal in measure, then the two lines are parallel.
- B-1-14 Master the property theorem of parallel lines: if two parallel lines are cut by a

transversal, then the corresponding angles and the alternate interior angles are respectively equal, and the same-side interior angles are supplementary.

B-1-15 Master the identification theorem of parallel lines: When two straight lines are cut by a transversal, if the alternate interior angles are equal or the same-side interior angles are supplementary, then the two lines are parallel.

B-1-16 Be able to draw a parallel line to a given straight line, passing through a point not on the line using a set square and a straightedge.

### **Learning category B-2: Triangle**

B-2-1 Understand the concept of triangles and their sides, interior angles, exterior angles, the median, altitude, and angle bisector; master the notation of triangles; understand the stability of a triangle.

B-2-2 Comprehend the concept of isosceles triangles, equilateral triangles, right triangles, acute triangles and obtuse triangles; know about their symbolic notation; be able to categorise triangles based on their side lengths and angle measures.

B2-3 Be able to explain the relationship between the three sides of the same triangle: the sum of the lengths of any two sides of the triangle is greater than the length of the third side.

B-2-4 Master the triangle sum theorem, as well as the property that an exterior angle of a triangle equals to the sum of the two non-adjacent interior angles.

B-2-5 Understand the concept of congruent triangles; be able to identify their corresponding sides and corresponding angles; know that their corresponding sides and corresponding angles are equal.

B-2-6 Understand the identification axiom of congruent triangles: two triangles are congruent if any pair of corresponding sides and their included angle are equal in both triangles; two triangles are congruent if any two angles and their included side are equal in both triangles; two triangles are congruent if all three sides in one triangle are congruent to the corresponding sides in the other. Understand the inference: two triangles are congruent if two angles and one of the sides opposite to one of the equal angles are congruent in both triangles.

B-2-7 Master the property theorem of isosceles triangles: the two base angles of an isosceles triangle are equal, the altitude, the median and the bisector of the vertex angle coincide. Understand the identification theorem of isosceles triangles: a triangle is isosceles if any of its two angles are equal.

B-2-8 Master the property theorem of equilateral triangles: all angles of an equilateral triangle measure 60 degrees. Know about the identification theorem of equilateral triangles: a triangle is equilateral if the three angles are exactly the same, or if it is an isosceles triangle with one angle measuring 60 degrees.

B-2-9 Master the properties of right triangles: the two acute angles of a right triangle are complementary; know that a triangle with two complementary angles is a right triangle.

B-2-10 Master the identification theorem of the congruence of right triangles: two right triangles are congruent if one pair of sides and the hypotenuse are equal.

- B-2-11 Master the Pythagorean Theorem and the Converse of Pythagorean Theorem.
- B-2-12 Be able to calculate, prove and solve some simple practical problems by using the Pythagorean Theorem and its Converse.
- B-2-13 Understand the ratio of line segments and the proportional segments; understand the golden ratio through real examples of architecture and arts.
- B-2-14 Understand the axiom: if two straight lines are cut by a group of parallel lines, the corresponding segments are proportional.
- B-2-15 Understand the concept of similar triangles; learn about the ratio of similitude.
- B-2-16 Understand the identification theorem of similar triangles: if two triangles have two of their angles equal, the triangles are similar; if two triangles have two pairs of sides in the same ratio and the included angles are also equal, then the triangles are similar; if two triangles have three pairs of sides in the same ratio, then the triangles are similar.
- B-2-17 Understand the property theorem of similar triangles: in two similar triangles, the ratio of the corresponding segments is equal to the similarity ratio; the ratio of perimeters is also equal to the similarity ratio.

### **Learning category B-3: Quadrilateral**

- B-3-1 Understand polygons and the related concept, including the vertices, sides, interior angles, exterior angles, diagonals etc., of a polygon; grasp the formulas for the sum of interior angles and that of the exterior angles of a polygon; be able to use them to perform calculations.
- B-3-2 Understand the concept of parallelograms, rectangles, rhombuses, squares, and trapeziums; be able to explain their subordinate relations and the difference between them.
- B-3-3 Understand the instability of quadrilaterals.
- B-3-4 Master the property and identification theorems of parallelograms, rectangles, rhombuses, and squares; be able to apply these theorems in solving related problems.
- B-3-5 Understand the meaning of the distance between two parallel lines; and be able to measure the distance between two parallel lines.
- B-3-6 Understand the meaning of triangle midpoint theorem.

### **Learning category B-4: Circle**

- B-4-1 Comprehend the concept of circle, arc, chord, central angle, and inscribed angle.
- B-4-2 Understand the theorem of constructing a circle through three given points.
- B-4-3 Master the vertical theorem and be able to use it to prove the related propositions.
- B-4-4 Know about the relationship between a central angle and its corresponding arc as well as the relationship between an inscribed angle and its corresponding arc; understand and prove the inscribed angle theorem as well as its inference: the measure of an inscribed angle is half of the measure of the central angle subtended by the same arc; the inscribed angle subtended by a diameter is a right angle; a  $90^\circ$  inscribed angle has a chord which is the

diameter of the circle; the opposite angles of a cyclic quadrilateral are supplementary.

- B-4-5 Understand the positional relationship between a point and a circle.
- B-4-6 Understand the positional relationship between a straight line and a circle; master the concept of tangents and tangent-chord angles; be able to construct the tangent to a circle at a given point on the circle with a set square.
- B-4-7 Master the identification and properties of tangents: a tangent to a circle is perpendicular to the radius at the point of tangency; on the contrast, a straight line perpendicular to a radius of a circle at its endpoint is a tangent line to the circle. Understand the tangent length theorem: the lengths of the two tangents drawn from an external point to a circle are equal; the line from the centre of a circle to this point bisects the angle between the two tangents. Be able to use the theorem to perform the related calculations and demonstrate proofs.
- B-4-8 Be able to calculate the length of an arc of a circle and the area of a sector.

### **Learning category B-5: Solid figure**

- B-5-1 Understand the surface development of right prism, cylinder and cone; be able to calculate their surface area.
- B-5-2 Master the formulas and methods for calculating the volume of a cone and be able to solve simple practical problems.
- B-5-3 Know the concept of central projection and parallel projection.

### **Learning category B-6: Trigonometric Function of Acute Angles**

- B-6-1 Understand trigonometric functions of acute angles such as  $\sin A$ ,  $\cos A$ ,  $\tan A$ ; be able to find the values of trigonometric function for angles that measure  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ .
- B-6-2 Be able to find the values of trigonometric function for all angles that measure between  $0^\circ$  and  $90^\circ$  with an electronic calculator.
- B-6-3 Be able to use trigonometric functions of acute angles to solve right triangles.
- B-6-4 Be able to solve the respective practical problems using the relationship between the sides and angles of a right triangle.

### **Learning category B-7: Coordinates and transformation**

- B-7-1 Understand the concept of the Cartesian coordinate system, origin, coordinate axis, abscissa, ordinate and quadrants.
- B-7-2 Be able to locate a point according to the given coordinates in a given Cartesian coordinate system; and to write the coordinates according to the position of the point.
- B-7-3 Recognise axial symmetric figures through concrete examples; understand the basic properties of axial symmetry: the line segment joining symmetric points is perpendicular to and bisected by the axis of symmetry.
- B-7-4 Know the symmetry properties of isosceles triangles, rectangles, rhombuses, and circles.

- B-7-5 Be able to draw some simple axial symmetrical figures in terms of points, line segments and triangles; recognise and appreciate the axial symmetrical figures in nature and real life.
- B-7-6 Recognise centrosymmetric figures through concrete examples; as well as the centrosymmetric property of parallelograms, rectangles, rhombuses, and circles.
- B-7-7 Be able to draw some simple centrosymmetric figures; recognise and appreciate the centrosymmetric figures in nature and real life.

**Learning category B-8: Compass-and-straightedge construction as well as geometric proof**

- B-8-1 Be able to perform the following basic construction with a straightedge and compass: construct a line segment equal to a given line segment; construct an angle equal to a given angle; construct a bisector of an angle; construct a perpendicular bisector of a line segment; construct a perpendicular line to a given line through a given point.
- B-8-2 Be able to construct triangles using the basic construction method: construct a triangle given the length of all three sides, two sides and its included angle, as well as two angles and its included side; construct an isosceles triangle given the base and the altitude on the base; construct a right triangle given a cathetus and the hypotenuse.
- B-8-3 Be able to construct a circle using the basic construction method: construct a circle through three non-collinear points.
- B-8-4 In compass-and-straightedge construction, understand the principles of the related construction, and keep the traces of construction; however, illustration of the construction process is not required.
- B-8-5 Understand the significance of definition, proposition, theorem and inference.
- B-8-6 Understand what a mathematical proof is as well as its necessity, procedures and forms.
- B-8-7 Be able to conduct inferences based on definition, axiom and theorem; be able to complete a geometric proof correctly.
- B-8-8 Have a primary understanding of the functions of counterexamples; know how to use counterexamples to prove that a proposition is false.
- B-8-9 Recognise the meaning of proof by contradiction through real examples.

**Learning domain C: Statistics and Probability**

**Learning category C-1: Statistics**

- C-1-1 Understand the basic statistical process in terms of sampling, data processing, data analysis, statistical inference.
- C-1-2 Know the necessity of sampling, realise the relationship between population and samples.
- C-1-3 Be able to draw statistical graphs like bar charts, broken line graphs, pie charts, etc., by hand or with computer software.

- C-1-4 Understand the significance of frequency; be able to make frequency distribution tables and histograms based on the given data and use them to explain the related practical issues.
- C-1-5 Master the concept of mean, median, mode, range, variance, and standard deviation; be able to calculate or use a calculator to find these statistical quantities to solve simple statistical problems.
- C-1-6 Understand the concept of weighted mean; be able to find the weighted mean for a set of data.
- C-1-7 Cultivate scientific attitude and enhance the awareness of supporting ideas with data in statistical activities.

### **Learning category C-2: Probability**

- C-2-1 Combining with concrete examples; realise the random phenomena; understand the meaning of the probability of simple random events.
- C-2-2 Be able to distinguish between inevitable events and random events; be able to use tabular methods and tree diagrams to calculate the probability of events.
- C-2-3 Understand that, through large number of repeated experiments, frequency can be used to estimate probability.

### **Learning category D: Emotions, attitudes, and values**

- D-1-1 Actively participate in mathematical activities, have curiosity and the desire to learn mathematics.
- D-1-2 Actively engage in such mathematical activities like observation, operation, induction, conjecture, verification, etc.; be able to express and exchange ideas on their own thinking process.
- D-1-3 Try to discover and raise mathematical questions in real life situations, use mathematical methods to analyse and solve problems.
- D-1-4 Through practical problem solving, realise the value of mathematics and increase interest in learning mathematics.
- D-1-5 Be able to express and exchange ideas about mathematics problems and methods in exploratory activities; learn to listen to and respect others' views.
- D-1-6 Be able to overcome difficulties encountered in mathematical problem-solving; enhance self-confidence in mathematics learning.
- D-1-7 Be able to realise the connection between mathematical knowledge.
- D-1-8 Be able to form the habit of thorough thinking and the attitude of seeking truth from facts in mathematical operations and reasoning.